Journal of the European Ceramic Society 17 (1997) 113-120
© 1996 Elsevier Science Limited
Printed in Great Britain. All rights reserved
0955-2219/97/$17.00
PII: S0955-2219(96)00154-9

Plane Cross-Section Method for Modelling the Pressing of Complex-Shape Powder Articles in Rigid Dies

E. Olevsky,[a] M. Shtern[b] & V. Skorohod[b]

[a]Katholieke Universiteit Leuven, MTM Department, de Croylaan 2, Leuven, B-3001, Belgium
[b]Institute for Problems of Materials Science, Krzhizhanovsky str. 3, Kiev, 252180, Ukraine

(Received 15 September 1995; accepted 23 July 1996)

Abstract

Densification processes in rigid dies are considered. The behaviour of the powder products with inclined, curve and stepped surfaces is studied. The influence of a pressing cyclogram on the distribution of residual porosity is investigated. The analysis is based on the plane cross-section method. In the capacity of the procedure, promoting an optimization of densification, the subdivision of an article into separated elements pressed by different punches is used. For components with inclined surfaces, on the basis of the density distribution required for the final product the most relevant methods of this subdivision are pointed out. © 1996 Elsevier Science Limited.

Notation

a	Dimension of the characteristic cross-section in the horizontal direction
e	Volume change rate (the first invariant of the strain rate tensor)
e_i	ith component of the strain rate tensor
h_{in}	Initial height of the nth part of a characteristic cross-section
h_{fn}	Final height of the nth part of a characteristic cross-section
l	Billet's size in the perpendicular to a characteristic cross-section direction
M	Mass of powder
t	Time
V_1	Lower punch velocity
V_2	Upper punch velocity
W	Equivalent strain rate
$y_1(x,t)$	Expression for the contour limiting the characteristic cross-section at the bottom
$y_2(x,t)$	Expression for the contour limiting the characteristic cross-section at the top
γ	Shape change rate (the second invariant of the strain rate deviator)
η^*	Generalized viscosity coefficient
ρ	Density of a powder volume
σ	Equivalent stress
σ_i	ith component of the stress tensor
φ, ψ	Functions of density ρ and being analogies of the shear and bulk viscosity moduli for porous material

Introduction

The finite[1-4] and permeable element methods[5-8] are dedicated for the solution of boundary-value problems concerning working by pressure of powder ceramic and metal components in order to optimize the conditions of shape forming and densification.

These methods in their expanded form are rather labour-consuming ones, requiring quite a long computation time. This lends impetus to the development of simpler and more pictorial methods of modelling based on the analysis of situations appearing in real technologies. It should be noted that possible simplified methods do not represent an alternative to global ones. Moreover, they can be used for control analysis accompanying the application of one of the above-mentioned methods.

The method of accelerated modelling, based on the hypothesis of plane cross-sections and being attendant to the permeable element method, is proposed in the present work.

Characteristic Cross-Section of a Complex-Shape Article: Cyclogram of Pressing

In order to optimize the production of a complex-shape article, the following procedure is used.

If the article has no symmetry elements, if it is not a body of rotation or a plane one, then such a cross-section is built, whose upper and lower boundaries have the largest deviation from the plane perpendicular to the pressing direction (Fig. 1).

The cross-section is subdivided into elements by lines which are parallel to the above-mentioned direction. These lines should correspond to the places with largest curvature of the article. If the article has steps, then the lines should separate them. The possibility of pressing each element of the volume by a separate punch, moving in accordance with a special law, is supposed. Such cross-sections are termed characteristic. Articles of particular complex shape can have several characteristic cross-sections. For bodies of rotation, the characteristic cross-section coincides with an axial one.

For the determination of the initial product configuration and pressing mode, consider only one of the characteristic cross-sections. The choice of the initial configuration is carried out on the ground of the requirement of proportionality of the heights of different elements of the initial product to the heights of the same elements for the final product after pressing. Therefore, if the article is subdivided by separating lines into N elements, and the kth element should have the final height h_{fk} and density ρ_{fk}, then its initial height is determined:

$$h_{ik} = \frac{h_{fk}\rho_{fk}}{\rho_i} \quad (1)$$

where ρ_i is a density of the initial product. Reasoning from a certain article shape, other principles of determination of the initial product's height are possible. In the above-mentioned formula (1), h_{fk} and h_{ik} represent an average height of the layer, if the contour of the corresponding element is not plane.

A pressing scheme is determined by functions $h_k(t)$ being varied in the interval $[h_i,h_f]$. A graphic representation of all such functions combined in one single frame of reference, where the process time is plotted as abscissas and the values of $h_i(t)$ as ordinates, is a cyclogram of pressing.

The procedure of building a cyclogram of pressing is demonstrated (Fig. 2) for the stepped article shown in Fig. 3.

With the availability of the relevant equipment, the scheme is realized, providing the proportionality of the velocities of movement of the punches, corresponding to the different elements of the article's volume, to the current heights of these elements. It will be shown below that this scheme provides uniformity of density distribution in the radial direction for stepped articles. In this case, the velocities of the punches are different, which is manifested by different slopes of the corresponding lines in Fig. 2.

If there is no possibility to provide the above-mentioned scheme with proportional punch movements, the scheme with consequent pauses can be used. Here, instead of the requirement of the proportionality of the punch velocities to their current axial coordinates, the condition of the proportionality of the initial and final billet heights is applied. However, in this case, all the punches move with an equal velocity, but not simultaneously.

The procedure of building a pressing cyclogram with consequent pauses consists of the following (Fig. 4). The frame of reference is built, where the conventional time of the process T is plotted as

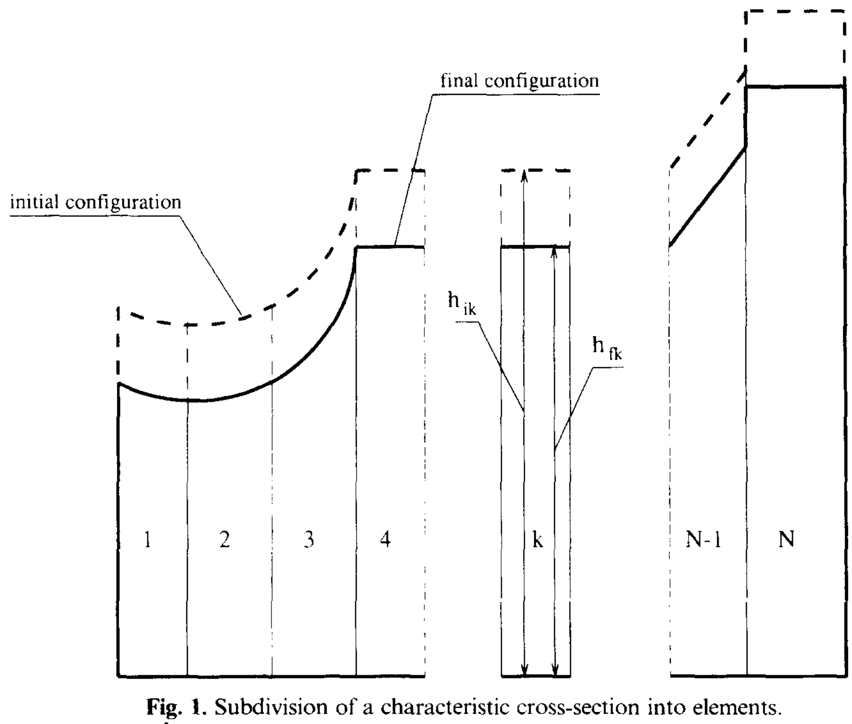

Fig. 1. Subdivision of a characteristic cross-section into elements.

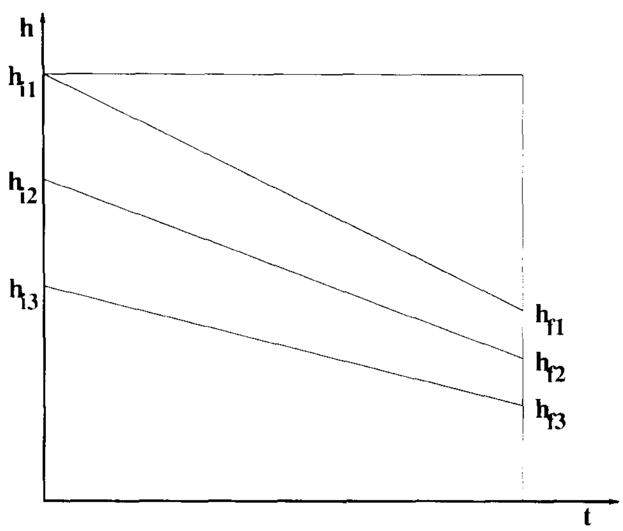

Fig. 2. Cyclogram of pressing,

abscissas, and the values of the element heights, as ordinates. Then, at the corresponding point on the abscissa, a perpendicular is erected, on which the values of the final heights of the article parts considered are plotted (Fig. 4). The initial height values, determined by formula (1), are plotted as ordinates. The points h_{in} and h_{fn}, corresponding to the initial and final heights of the article's nth part pressed without a pause, are connected by a straight line. At each point h_{fj} a straight line is built, which is parallel to the line h_{in}–h_{fn} until it intersects with a line built at h_{ij} and being parallel to the abscissas.

A parallelism of the inclined segments testifies that the velocities of the movement of corresponding punches are equal. Along with this each punch starts the movement after the nth punch passes some definite ordinate. Its value is determined as a coordinate of the vertex point of the cyclogram.

Flow under Plane Deformation: Semi-analytical Solution for the Field of Velocities

The analysis of the flow of densified material in a characteristic cross-section is carried out based on the assumption that the velocity component in the direction, which is perpendicular to the plane of the characteristic cross-section, is much smaller than other components and its value can be identified with zero. This assumption permits the consideration of the evolution of the characteristic cross-section state in the framework of plane deformation ideas.

The analysis is carried out, using the expressions:

$$\gamma_1 = \sqrt{(e_x - e_y)^2 + 4e_{xy}^2}$$
$$\tau_1 = \tfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}, \quad p_1 = \tfrac{1}{2}(\sigma_x + \sigma_y) \tag{2}$$

Then the equivalent strain rate and stress can be expressed as follows:

$$W = \frac{1}{\sqrt{1-\theta}}\sqrt{\frac{1}{2}\varphi\gamma^2 + \left(\frac{1}{6}\varphi + \psi\right)e^2}$$
$$\sigma = \frac{1}{\sqrt{1-\theta}}\sqrt{\frac{\gamma_1^2}{\frac{1}{2}\varphi} + \frac{p_1^2}{\psi + \frac{1}{2}\varphi}} \tag{3}$$

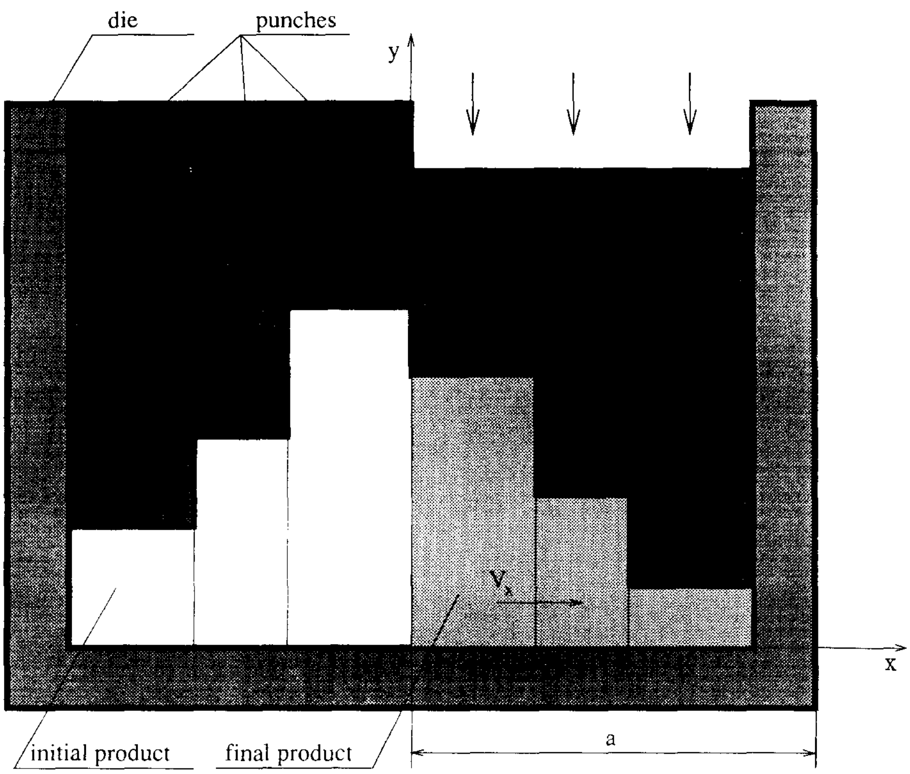

Fig. 3. Schematic of pressing of a stepped article.

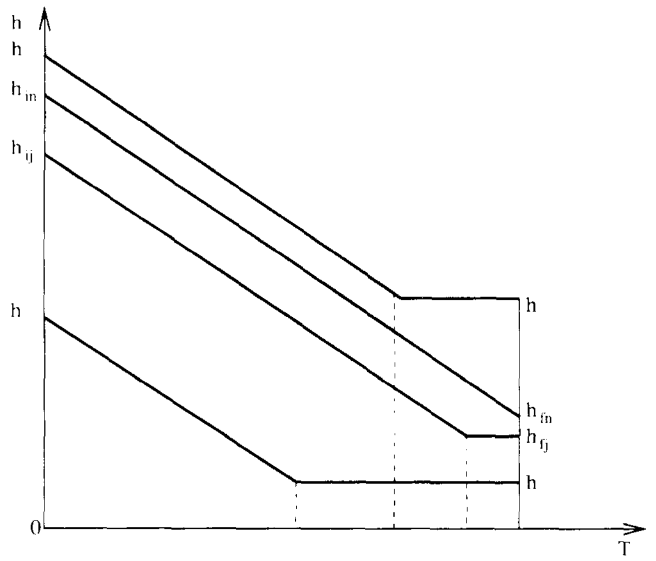

Fig. 4. A pressing cyclogram.

and the scalar constitutive equations will be:

$$p_1 = \frac{\sigma}{W}\left(\frac{1}{6}\psi + \varphi\right)e \quad (4)$$

$$\tau_1 = \frac{1}{2}\frac{\sigma}{W}\varphi\gamma_1 \quad (5)$$

Stress and strain rate components are connected by the relationships:

$$\sigma_x = \frac{\sigma}{W}\left(\psi + \frac{2}{3}\varphi\right)e_x + \left(\psi - \frac{1}{3}\varphi\right)e_y) \quad (6)$$

$$\sigma_y = \frac{\sigma}{W}\left(\psi + \frac{2}{3}\varphi\right)e_y + \left(\psi - \frac{1}{3}\varphi\right)e_x) \quad (7)$$

$$\tau_{xy} = \frac{\sigma}{W}\varphi e_{xy} \quad (8)$$

The equations of quasistatics have the form:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$
$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \quad (9)$$

The plane cross-sections method is based on the supposition that the axial component velocity V_y is a linear function of y, and the perpendicular component V_x does not depend on y. The foregoing means that the vertical cross-sections are kept vertical even after deformation.

Let

$$y = y_1(x,t); \qquad y = y_2(x,t) \quad (10)$$

be the equations of the parts of a contour limiting the characteristic cross-section at the bottom and at the top, respectively.

The use of time as an argument of both functions characterizes a possibility of changing the contour with time both owing to the punches' displacement as a whole and on account of the displacements of separated punches in the case when an autonomous movement of the press-elements is possible.

Let $V_2(x,t)$ and $V_1(x,t)$ be the velocities of the upper and the lower punches, respectively. Then, due to the hypotheses of the plane cross-section method:

$$V_y = \frac{y_2 - y}{y_2 - y_1}V_1 + \frac{y - y_1}{y_2 - y_1}V_2 \quad (11)$$

and

$$e_y = \frac{V_2 - V_1}{y_2 - y_1} \quad (12)$$

where e_y is the strain rate in the direction y.

Thus, the axial components of the displacement and deformation rates turn out to be known. They are completely determined by the functions y_1, y_2, V_1, V_2, which, in turn, are calculated on the basis of a pressing cyclogram.

The shear component of the strain rate is also known:

$$e_{xy} = \frac{1}{2}\frac{\partial V_y}{\partial x} \quad (13)$$

The transfusion velocity V_x as well as the density ρ are the unknown sought values.

For pressing in rigid dies, the kinematic boundary conditions:

$$V_x|_{x=0} = V_x|_{x=a} = 0 \quad (14)$$

The plane cross-section method gives the following expression for the velocity component V_x (due to the solution of eqns (9) taking into consideration eqns (2)–(13) and the boundary conditions (14)):

$$V_x = -\int_0^x \frac{\psi + \frac{1}{6}\varphi}{\psi + \frac{2}{3}\varphi}e_y\mathrm{d}x + \frac{x}{a}\int_0^a \frac{\psi + \frac{1}{6}\varphi}{\psi + \frac{2}{3}\varphi}e_y\mathrm{d}x -$$
$$\int_0^x \frac{1}{4\eta^*(\psi + \frac{2}{3}\varphi)}\int_0^{x_1} e_y\frac{\partial}{\partial x_1}(\eta^*\varphi)\mathrm{d}x_2\mathrm{d}x_1 \quad (15)$$
$$+ \frac{x}{a}\int_0^a \frac{1}{2\eta^*(\psi + \frac{2}{3}\varphi)}\int_0^{x_1} e_y\frac{\partial}{\partial x_2}(\eta^*\varphi)\mathrm{d}x_2\mathrm{d}x_1$$

where φ and ψ are the functions of density ρ, being the analogies of the shear and bulk viscosity moduli for porous material; η^* is the generalized viscosity coefficient, $\eta^* = \frac{\sigma}{2W}$; a is the dimension of the characteristic cross-section in the horizontal direction.

The analysis of expression (15) shows that V_x depends on the deformation scheme (e_y), on the current density distribution (φ and ψ), and on the rheology of the matrix phase (η^*). In order to determine the influence of the rheological properties, the second item in the right-hand part of eqn (15) can be considered. It follows from (6) that the product $\eta^*(\psi + \frac{2}{3}\varphi)$ is proportional to the axial stress and it

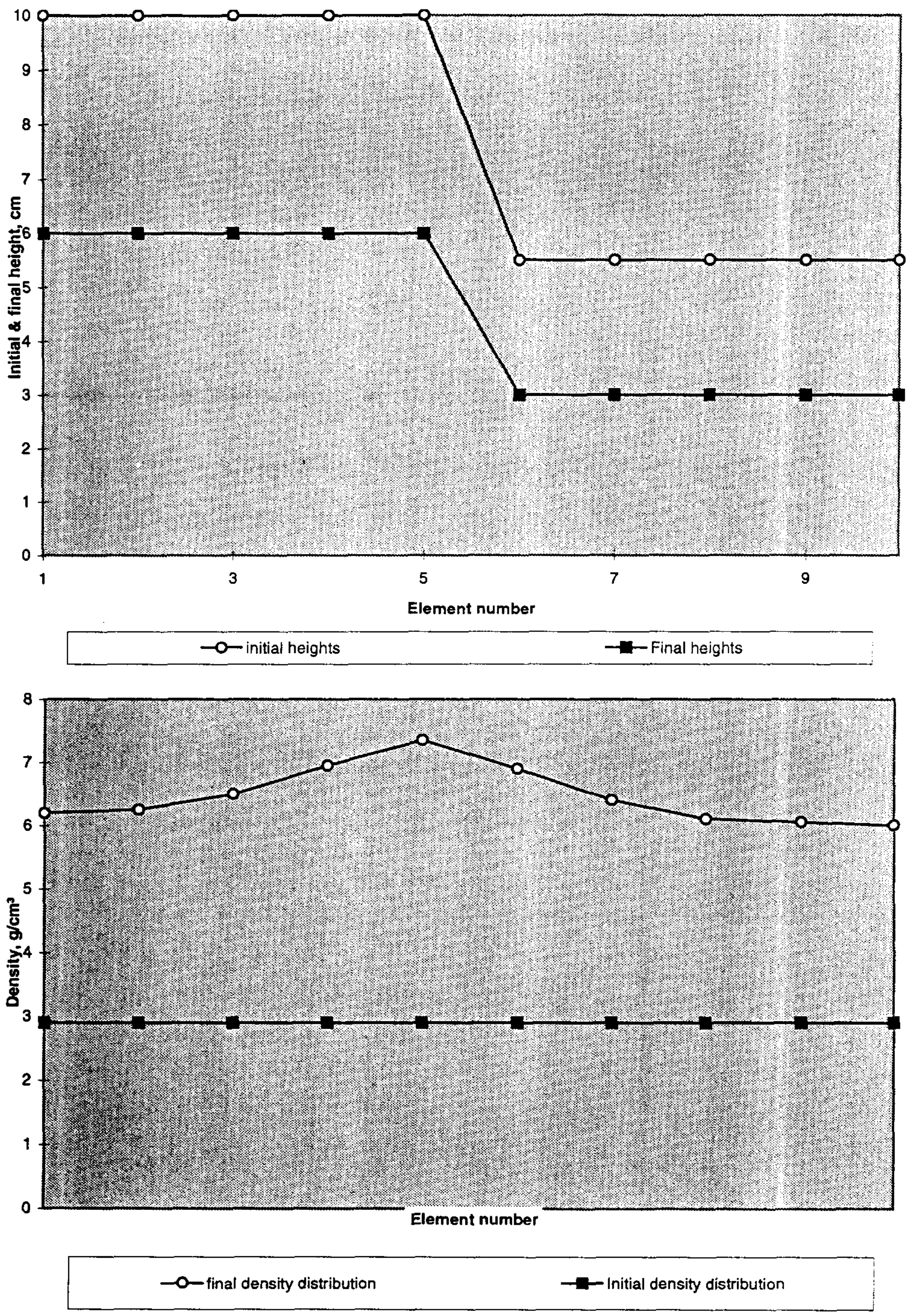

Fig. 5. Calculation results for pressing the stepped article by split punches.

follows from (8) that $\eta^*\varphi$ is proportional to the tangential stress. The latter is determined by the friction resistance of the vertical powder layers.

The stress caused by the internal friction is much smaller than the externally applied one. Therefore, the item, whose order of magnitude is $\frac{1}{2}\frac{\tau_{xy}}{\sigma_y}$, is much smaller than the first item in the right-hand part of eqn (15).

From the foregoing it follows that the rheological properties of the solid phase have no significant influence on the transfusion velocity.

Also, the analysis of eqn (15) shows that the powder transfusion has a place when the product $e_y\frac{\psi+\frac{1}{6}\varphi}{\psi+\frac{2}{3}\varphi}$ changes going from one cross-section to another.

The Continuity Equation for the Plane Cross-Section Method

Let M be the mass of powder, enclosed by the left boundary of the article and an arbitrary cross-section x. Then:

$$M(x,t) = l\int_0^x \rho h(x,t)\mathrm{d}x \qquad (16)$$

where l is a billet's size in the perpendicular to the characteristic cross-section direction, and $h(x, t) = y_2 - y_1$.

In accordance with the mass conservation law:

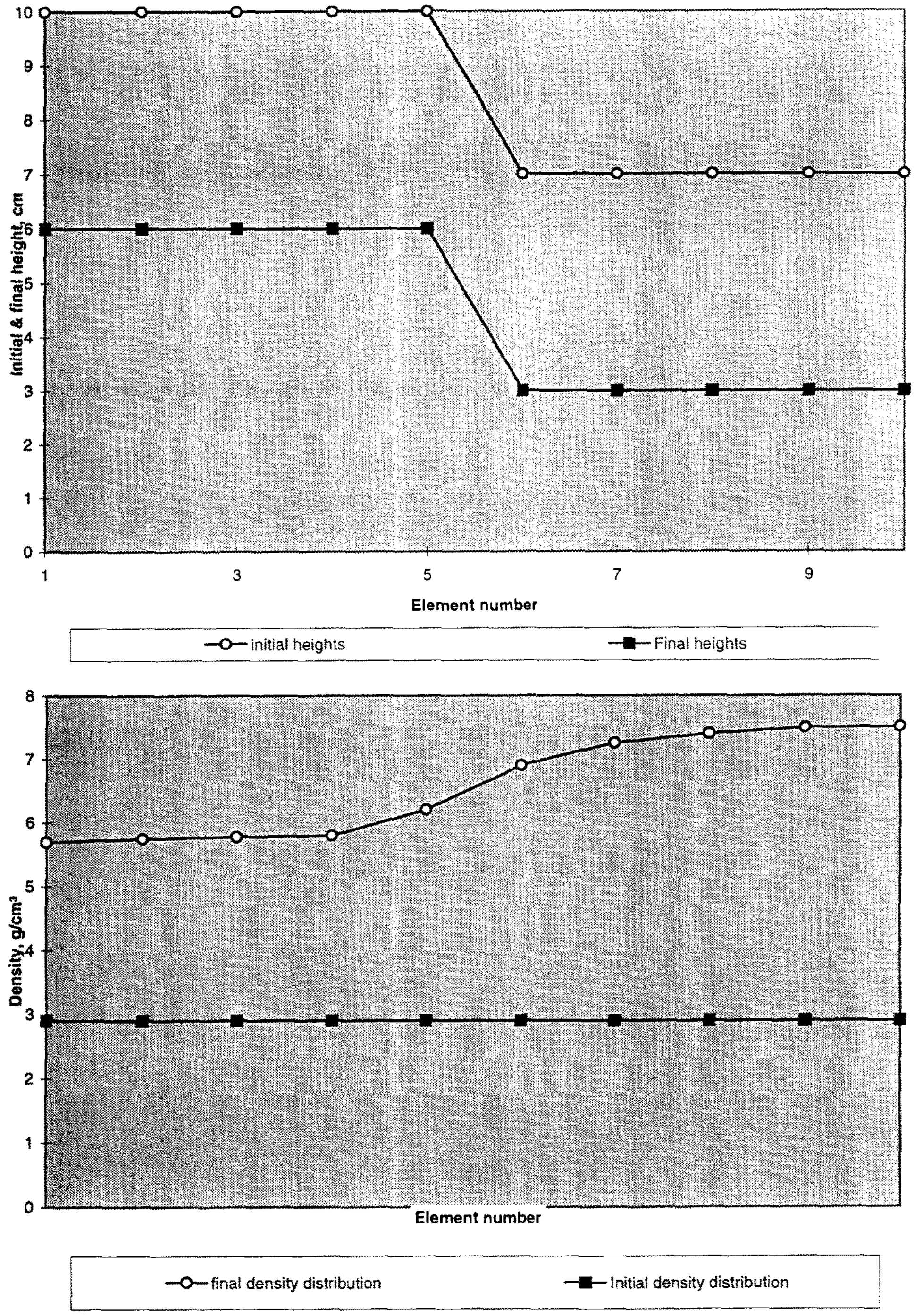

Fig. 6. Calculation results for pressing the stepped article by a single punch.

$$\frac{\partial M}{\partial t} = -V_x \rho h \quad (17)$$

Equation (17) can be represented in the following form:

$$\frac{\partial}{\partial t}(\rho h) + V_x \frac{\partial}{\partial t}(\rho h) + \rho h \frac{\partial V_x}{\partial x} = 0 \quad (18)$$

In general, (18) can be considered as a differential equation relatively ρh. Its solution can be obtained, if V_x is known.

Thus, the combined solution of eqns (15) and (18) permits the determination of the transfusion velocities and distribution of residual density.

The Condition of the Attainment of a Uniform Density Distribution

The problem of powder transfusion is connected with another important problem — the possibility of provision at each time moment of the completely controlled and, in particular, uniform versus x density distribution. It turns out that its solution is attained only in absence of the transfusion flow. Indeed, in this case, it follows from (18) that

$$\rho(x,t) = \frac{\rho_i(x) h_i(x)}{h(x,t)} \quad (19)$$

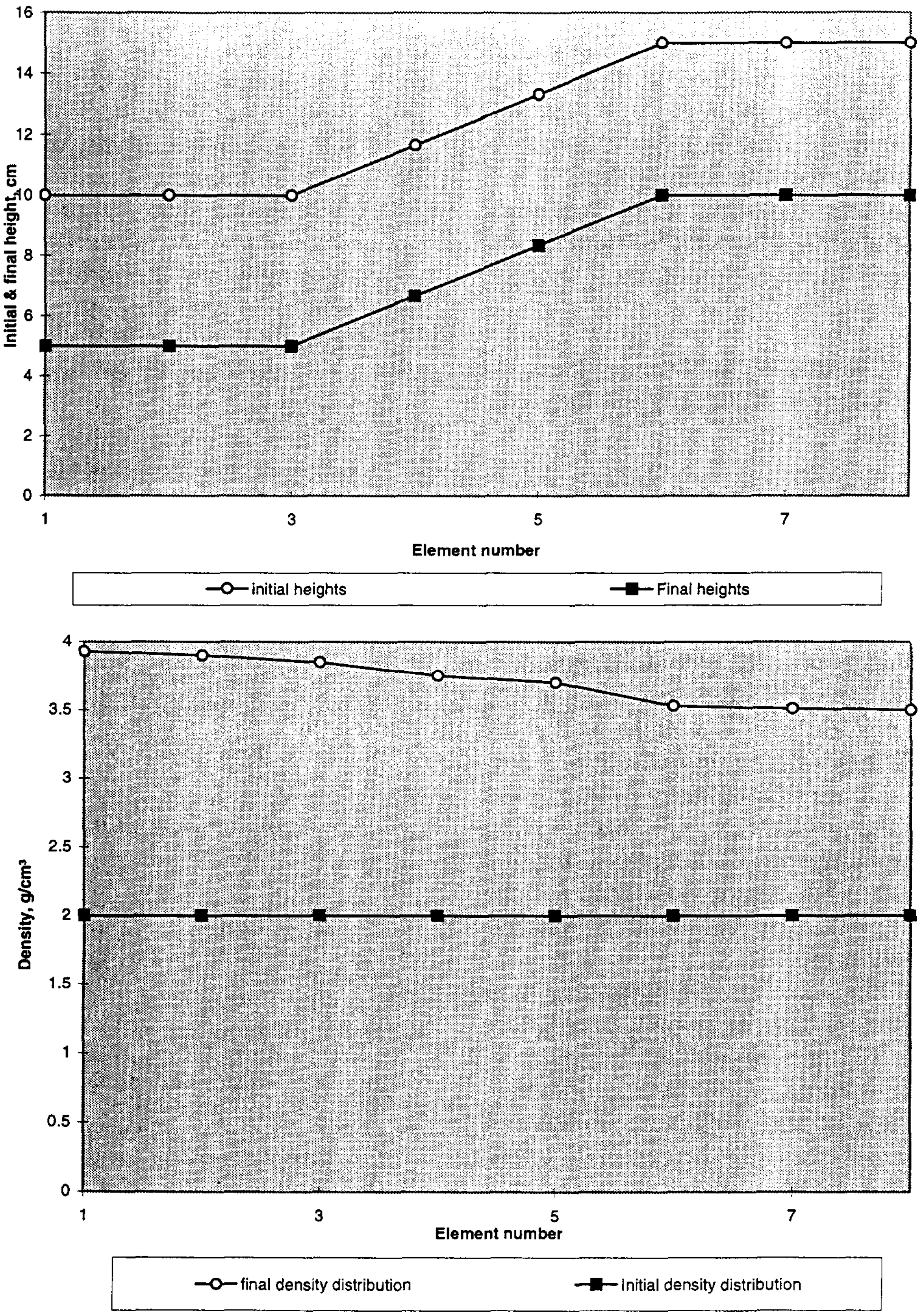

Fig. 7. Calculation results for pressing the article with a stepped-inclined surface.

Choosing corresponding $\rho_i(x)$, $h_i(x)$ and $h(x,t)$, any predetermined value of $\rho(x,t)$ can be obtained. However, for an arbitrary $\rho_i(x)$ the deformation scheme should be chosen such that at any moment of time the product $y\frac{1}{h}\frac{\partial t}{\partial h}$ does not depend on x. This is possible only under correction of the pressing scheme by a 'feed back' principle: on the basis of the density distribution control.

This procedure is simplified if $\rho_i(x)$ is a constant value. Then for the pressing scheme, providing:

$$h(x, t) = h_i(x)f(t) \tag{20}$$

where $f(t)$ is some arbitrary function of time, density is a constant at each moment of time. This condition also provides a zero value of the transfusion velocity. The result obtained can be formulated as following: for production of the complex-shape article with density distributed uniformly at each moment of pressing, the initial density should be distributed also uniformly and axial deformation velocities of different elements of the article's volume should be proportional to the current heights of these elements.

The above-mentioned statement is real only for articles with stepped surfaces. For such shape of an article, the subdivision of the article's volume into elements with uniform internal distribution of density, which are pressed by different punches, is possible. If the article is limited by a curve or an inclined surface, this method of pressing does not provide a totally uniform density distribution. Even the use of the split punches causes a density heterogeneity by virtue of the fact that the boundary contour of some elements is not plane.

The solution of these problems, however, cannot be obtained by semi-analytical methods. Here it turns out to be necessary to solve the problem numerically.

Residual Density Distribution for Pressing in a Rigid Die of the Article with Stepped and Inclined Sections of Surface

The results of the calculation of the density distribution in the stepped article for pressing by split punches (for high and low parts of the article) and by a single punch are represented in Figs 5 and 6, respectively.

Because of the larger volume deformation, density of the lower part is higher than that of the lower one in the case of pressing of the articles by the single punch (Fig. 6). The results in Fig. 5 correspond to pressing with preliminary pause for the punch for the higher part. For this deformation scheme, the displacement of the punch for the higher part is 1·6 times more than that for the lower part. This pressing scheme permits symmetrical density distribution.

The results of the calculation for pressing by a single punch in the rigid die of alumina powder are shown in Fig. 7. The pressed article has a stepped-inclined surface. Due to a larger volume deformation, the lower part has the higher density. Along with this, its density decreases from the periphery to the centre of the article because of the material transfusion into the central part having the smaller density.

Conclusions

1. The plane cross-section method, dedicated for accelerated modelling of die-wall compaction of complex-shape powder articles, is discussed.
2. The character of the dependence of the transfusion velocities of the deformation scheme, current density distribution and rheology of the matrix phase is analysed.
3. It is shown that the proportionality of the velocities of movement of the punches, corresponding to the different elements of the article's volume, to the current heights of these elements provides uniformity of density distribution in radial direction for stepped articles.
4. Certain technological problems of pressing in rigid dies of complex-shape articles are solved.

References

1. Zienkiewicsz, O. C., *The Finite Element Method*, 2nd edn. McGraw-Hill, UK, 1977.
2. Lewis, R. W. & Schrefler, B. A., *The Finite Element Method in the Deformation and Consolidation of Porous Media*. Wiley, New York, 1987.
3. Abouaf, M. *et al.*, Finite element simulation of hot isostatic pressing of metal powders. *Int. J. Numer. Methods Eng.*, **25** (1988) 191–212.
4. Maximenko, A. L. *et al.*, Compacting of complex-form powder details by isostatic pressing of porous billets with density nonuniformity. In *Hot Isostatic Pressing 1993*, eds L. Delaey & H. Tas. Elsevier Science Amsterdam, 1994, pp. 61–67.
5. Olevsky, E. A. & Shtern, M. B., The calculations of pressing and forging powder materials by the permeable element method. In *New Powder Materials and Technologies in Machine-Building*, IPMS NAS Ukraine, Kiev, 1988, pp. 27–31 (in Russian).
6. Olevsky, E. A. *et al.*, Determination of the density field in the pressing of parts of complex shape by the permeable element method. *Sov. Powder Metallurgy*, No. 3, 1989, pp. 15–21.
7. Shtern, M. B. *et al.*, Use of split punches in the production of flanged parts from powders. Theoretical analysis. *Sov. Powder Metallurgy*, No. 4, 1989, pp. 26–31.
8. Olevsky, E., Timmermans, G., Shtern, M., Froyen, L. & Delaey L., The permeable element method for modelling of deformation processes in porous and powder materials: theoretical basis and checking by experiments, submitted to *Acta Met*.